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Composite materials: Inelastic behavior, damage, fatigue and fracture

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Abstract

Contributions of solid mechanics research to the development of composite materials and structures are reviewed. The main topics include the tensile and compressive strength of fibrous composites; transformation and residual fields in composite and laminated microstructures; plasticity and viscoplasticity of composites during processing, under thermomechanical service loads, and in the presence of evolving fatigue damage; delamination, damage and fracture; and future research needs.

1. Introduction

Few, if any, accomplishments of solid mechanics research in the second half of this century can match those associated with modeling of heterogeneous solids, in support of the development of composite materials and their use in numerous structural applications. From their early introduction in high performance aircraft and spacecraft, electronic packaging, and thermal management, boats and sports equipment, to their recent adoption in ship and marine construction, oil exploration and production, bridges, and road and rail vehicles, composite and sandwich structures have not only replaced more traditional materials, but enabled production of entirely new devices and structures. Penetration of composites into all areas of technology is well illustrated by the annual U.S. consumption rate of about 3 billion pounds of polymers reinforced by glass fibers.

Overall progress in development of more efficient, durable and environmentally friendly devices and products depends on availability of materials with physical properties that are often not available in nature. Therefore, the general area of material modeling that now covers all material systems and

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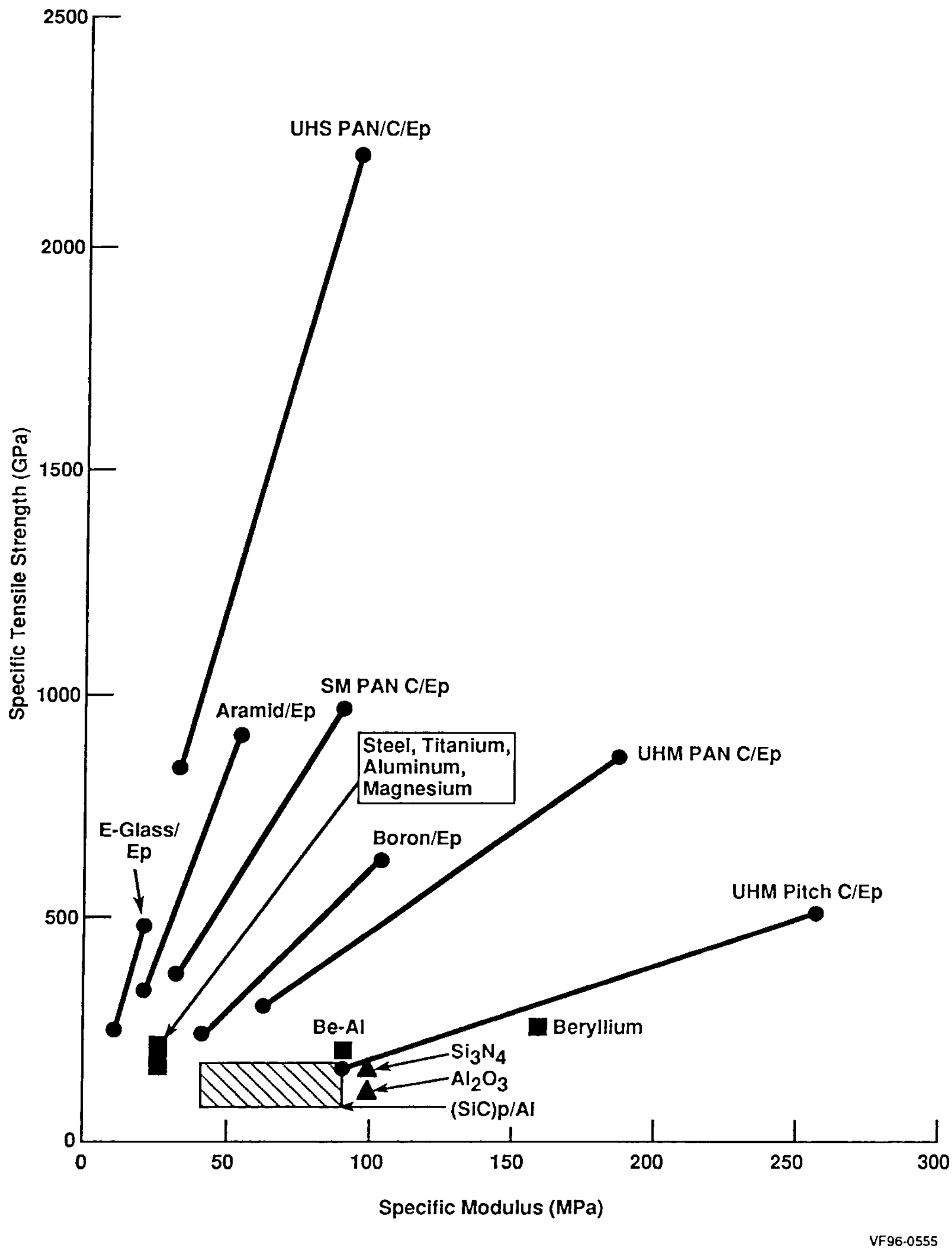

Fig. 1. Comparison of specific strength (strength/density) and specific longitudinal elastic moduli (modulus/ density) of structural composites and monolithic metals. (Courtesy of Dr. Carl Zweben.)

configurations, from the atomic and molecular to the micro, meso and macroscales, dominates solid mechanics research in publications, technical meetings, and, of course, in this volume.

Among the many subjects in micromechanics, this chapter outlines some recent results on the properties and behavior of composites reinforced by aligned fibers, and the laminated plates, shells and other parts made of composites. Since elastic response of such systems is reasonably well understood, our main focus is on problems that arise during loading beyond the purely elastic response of an undamaged material. The first two sections introduce some notable strength properties of typical polymer matrix systems, and discuss recent results on analysis of local stress and strain fields that are caused in these materials by sources unrelated to mechanical loading. Next, modeling of systems with metal matrices that can undergo significant inelastic straining is discussed, both under monotonic and cyclic loading. This ushers in the subjects of damage evolution, delamination and macroscopic fracture, which is of major concern in all composite systems. A listing of some research challenges closes the chapter. Very limited attention is accorded to evaluation of elastic moduli and local stress and strain fields during elastic response; these issues are mostly well understood and described in several papers and books (Hashin, 1972; Christensen, 1979, Hill, 1963, Nemat-Nasser and Hori, 1993, Reddy, 1997, Torquato, 1991; Willis, 1981). Moreover, some promising new directions in this area are discussed in the chapter by Torquato.

2. Tensile and compressive strength of fibrous composites

It is well known that fibrous composites have a fairly high longitudinal tensile strength and stiffness, comparable or superior to those of structural metals. When normalized by density, the strengths and moduli can be far superior. Fig. 1 compares these specific properties of several structural metals and two engineering ceramics, silicon nitride and alumina, with those of selected composite materials. The epoxy matrix systems are reinforced with aramid, boron, E-glass, and carbon fibers made from several precursor materials, polyacrilonitrile (PAN) and petroleum or coal tar pitch. Several types of carbon fibers are represented, standard modulus (SM), ultrahigh strength (UHS) and ultrahigh modulus (UHM) carbon-graphite fibers. The aluminum matrix systems are reinforced with boron fiber or silicon carbide particles. The comparison shows order of magnitude improvements in specific longitudinal strength and stiffness, derived from fiber properties, which motivate the dramatic influence of composite materials in aerospace and other engineering applications. Extensive research conducted since the 1960s has established several load sharing and transfer mechanisms that relate fiber strength and matrix properties to composite strength, as well as phenomenological failure criteria for fibrous plies and laminates (Zweben and Rosen, 1970, Harlow and Phoenix, 1978, Hashin, 1980, Hashin, 1985).

Evident from everyday experience is the contrast between tensile and compressive strength of unconstrained fibers and fabrics, which tends to assert itself in response of fibrous composite systems and laminates loaded in compression. This subject is of major importance due to the increasing use of thick composites which support large compressive stresses, e.g., in marine or civil infrastructure applications. Fiber waviness and misalignment had been identified early on as the main source of compressive strength and moduli degradation. More recent work has established that the unnotched compressive strength is governed by imperfection-sensitive microbuckling of the fibers (Budiansky and Fleck, 1993). Microbuckle and kink bands initiate at a region of misaligned or wavy fibers, and extend on planes whose normal is inclined at a certain angle, typically 20–30 degrees, to the fiber direction. Displacement measurements across growing microbuckles show a similarity with shear cracks, in that the material on one side of the band slides over the material on the other side. This failure mode has also been observed in laminates, where microbuckle bands can tunnel through the most axially compressed plies (Fleck et al., 1997). Many other kink band behavior characteristics have been recorded by Kyriakides et al. (1995) and analyzed by Schapery (1995).

The reduction of compressive strength and modulus depends on the degree of fiber waviness; since this feature is not easily controlled or determined *a posteriori*, the observed compressive strength values show substantial variation in nominally similar materials (Daniel et al., 1996; Hsiao and Daniel, 1996). In systems with large fiber misalignments, the compressive strength tends to level off at almost constant values, equal to about 0.1 to 0.2 G, the composite longitudinal shear modulus; this is also typical compressive strength of woven composites (Fleck, 1997). Open holes and notches reduce compressive strength by a similar amount. On the other hand, carefully fabricated plies or laminates with minimal misalignment have relatively high compressive strength. For example, Daniel and Ishaii (1994) report that thick AS4/3501-6 carbon/epoxy samples, carefully processed for minimal misalignment, sustain 1440 MPa compression, as compared to 2280 MPa in tension. Christensen and DeTeresa (1997) report even higher strength (1770 MPa) for a similar composite made of highly collimated cylindrical rods, where the fiber misalignment angle was less than 1 degree; this contrasts with the 560 MPa strength at misalignment angle of 4 degrees in the same material.

Fiber misalignment is less frequent in composites reinforced by larger diameter fibers ($\sim$150 μm), such as boron or silicon carbide, used primarily in metal, intermetallic and ceramic matrix systems, which are often carefully aligned prior to matrix deposition. The compression strength of such systems often exceeds the tensile strength, and reaches values in the 1500–3000 MPa range.

Fabrication methods promoting high fiber alignment are needed for improvement of compressive strength of laminated composite structures (Kempner and Hahn, 1995). In addition to control of the curing process, filament winding and fiber placement appear to be most adaptable in this regard because they allow application of fiber prestress for reduction of fiber waviness prior to matrix cure. The forces needed to reach required prestress magnitudes appear to be within the capacity of available equipment, for example, the prestress force of about 120 lbs imparts 1000 MPa prestress in a typical 10,000 filament tow of small-diameter glass or carbon fibers ($\sim$10 μm). Moreover, fiber prestress induces residual stresses into the composite structure, approximately equal to those caused by equivalent compressive tractions applied in the fiber direction to the prestressed plies. If properly selected, fiber prestress may retard transverse cracking, by causing transverse compression in the plies. However, detrimental residual stresses can be caused by an indiscriminate application of prestress, with the sole objective of reducing waviness. For example, Srinivas et al. (1999) have shown that a uniform fiber prestress applied in the wall of a cylindrical laminate generates residual stress gradients that in superposition with external or internal pressure would impair overall strength. An optimized distribution of the prestress forces through wall thickness resulted in minimal residual stresses while preserving high prestress magnitudes; however, the prestress magnitude depends on mandrel stiffness. Another application of fiber prestress is in counteracting the mechanical and thermal stresses that may cause free edge delamination in laminated plates.

The role of fiber prestress appears to be an unexplored aspect of mechanics of composite materials and laminates with a significant benefits potential. Modeling of all fabrication processes is needed, especially for thick structures under compression, where residual thermal stresses may cause extensive initial fiber waviness and kinking that have been observed, for example, to seriously degrade overall strength of submerged cylindrical laminates.

Buckling leading to possible delamination and compressive failure of thick laminated plates and shells is another serious issue that has been studied by Kardomateas and Philobos (1995) and other associates. They found critical load combinations for orthotropic shells subjected to axial and radial pressure, and quantified imperfection sensitivity leading to significant reductions in shell buckling strengths. Many other issues related to mechanics of thick composite structures are discussed in Rajapakse (1993).

3. Transformation and residual fields

Together with the stresses imposed by mechanical loads, composite materials and laminates must support residual stresses due to eigenstrains or transformation strains in the constituents and plies. These are strains not supported by a stress field; for example, thermal strains, moisture absorption strains, inelastic deformation or phase transformation strains are stress-free strains, also called eigenstrains or transformation strains. Fiber prestress can also be modeled in this manner. When present, these strains can be variable and dissimilar in constituent phases and plies, and are not by themselves compatible. For example, if a composite or polycrystal were disassembled and the phases or single crystal grains subjected to a uniform thermal change, they would develop thermal strains that would prevent their reassembly without application of additional tractions. Of course, compatibility of the total strains is required for material and structural integrity; it is enforced by residual elastic strains that create the residual stresses. Their magnitudes vary, but may be quite substantial, often exceeding those induced by allowable mechanical loading. This is of particular concern in ceramic and metal matrix systems processed at and cooled from high temperatures, but also in polymer matrix composites exposed to moisture absorption which may degrade matrix and even fiber elastic moduli. Therefore, either alone or in superposition with the mechanical stresses, the residual fields may cause internal damage, e.g., fiber-matrix interface decohesion, or matrix radial cracking, transverse cracking and free edge debonding of fibrous plies, and thus related changes in elastic moduli, strength, and in other physical properties, such as conductivity and thermal expansion (Lu and Hutchinson, 1995).

Ever since the classical work by Eshelby (1957), transformation strains or eigenstrains have had special significance in micromechanics of heterogeneous media. Eshelby had shown that a uniform transformation strain applied within an ellipsoidal region or inclusion in an infinite body creates a uniform strain field within that region. He also pointed out that an equivalent value of this eigenstrain can be found such that the stress and strain fields inside and outside the region are equal to those caused in and around an ellipsoidal inhomogeneity by a remote uniform field. Many problems associated with ellipsoidal inhomogeneities, cavities, cracks and other defects in solids have been analyzed with the help of equivalent eigenstrains (Mura, 1987; Zaoui and Masson, 1998).

In contrast to the extensive literature on evaluation of local fields caused by mechanical loading, simple solutions of elasticity problems for prescribed local eigenstrains in general microstructural geometries have been made possible only recently, by the discovery of several classes of uniform strain fields, which allow conversion of the eigenstrain problems into purely mechanical loading problems. In aligned fiber systems of arbitrary transverse geometry, and in multiphase aggregates of any geometry, where both stresses and strains can be made uniform, such uniform strain fields were first found by Dvorak (1982, 1990), analyzed further by Dvorak and Benveniste (1992), and also identified in laminated plates (Bahei-El-Din, 1992). The uniform fields are created in a representative volume of the heterogeneous solid by a superposition of the prescribed piecewise uniform phase eigenstrains with strains generated by application of an auxiliary overall uniform strain or stress. Since the fields are uniform, they do not depend on the geometry of the aggregate, and thus represent an exact elasticity solution. Of course, the auxiliary overall field needs to be removed, this may be accomplished with a standard micromechanical method. Superposing the solution of this purely mechanical loading problem with the uniform field provides the solution of the original eigenstrain problem.

Many applications of the procedure have been identified; for example, several universal connections between local and overall physical properties, and local stress and strain fields have been found in elastic an piezoelectric fibrous systems (Benveniste, 1993, 1996; Dunn, 1993; Dvorak and Benveniste, 1997; Chen, 1998). The transformation field analysis for incremental evolution of inelastic strains and/or damage under overall thermomechanical loading has also been developed in this context. In general, many physical processes, including constrained formation of distributed damage, can be described by certain equivalent eigenstrains applied to the original elastic material or structure. This provides a powerful tool for analysis of interactions of these processes, and for hierarchical modeling of their evolution on several size scales.

4. Plasticity and viscoplasticity

4.1. Initial stresses

Aluminum, titanium, magnesium, nickel or copper alloys, among others, as well as certain superalloys and intermetallics, can be reinforced by metal or ceramic fibers or particles for improved stiffness and strength (Chawla, 1987). Since the coefficients of thermal expansion of the constituent phases are usually quite dissimilar, cooling from the high fabrication and processing temperatures to room temperature almost always generates high enough thermal stresses for onset of local viscoplastic deformation. Additional inelastic deformation may be generated by mechanical pressure applied during fabrication. High residual stresses are therefore present in the as-fabricated state. They cannot be completely relieved by any thermal treatment, however, they can be modified in aligned fiber systems by application of isostatic or transverse pressure during cooling, and by adjustment of cooling rate (Yeh and Krempl, 1993; Jeong et al., 1994; Bahei-El-Din and Dvorak, 1995). Of course, response under mechanical loading at room or other operating temperatures is influenced by the residual fields, in fact, for certain loading directions, such loading may continue the inelastic deformation imposed by fabrication and cooling.

4.2. Matrix constitutive relations

The microstructural geometry of composite materials, with typical constituent sizes or reinforcement 'diameters' of the order of 1–100 micrometers, restricts deformation of the phases to very small and constrained volumes. This is most pronounced in metal-ceramic systems, where the reinforcement remains elastic under most loading conditions and, if the phases are well bonded, all inelastic deformation is concentrated in the matrix. Oxidation products, epitaxial layers, or coatings at the fiber-matrix interfaces may enhance or diminish this effect. In weakly bonded or partially debonded systems, displacement discontinuities caused by interface separation and sliding may contribute to certain apparently inelastic macroscopic deformation modes. Since the phenomenological inelastic constitutive relations for metals, polymers and other materials, and their experimental verifications, apply to relatively large and unconstrained material samples, they do not necessarily hold in the very small and constrained microstructures of composite materials, where entirely different dislocation patterns may develop during processing and cooling, and also during isothermal inelastic straining. Some of these issues in metal composites are addressed by Arsenault (1991) and Clyne and Withers (1993) and Taya and Arsenault (1989), among others, and also by J. W. Hutchinson in this volume. However, in the absence of reliable dislocation-based theories, the phenomenological constitutive relations are typically used in micromechanical analysis, albeit with modified material parameters, preferably derived from selected experiments on specific matrices and/or composite systems.

4.3. Yield surfaces and hardening of fibrous systems

While the microstructural constraints challenge the validity of constitutive theories, they may prefer certain local deformation mechanisms that simplify prediction of some aspects of the overall response. This is apparently the case in metal matrix composites reinforced by stiff, aligned elastic fibers; particularly so in the boron/aluminum system, and by analogy also in silicon carbide/aluminum and silicon carbide/titanium. Such fibers limit operation of plastic slip systems on planes that intersect the fiber, thus restricting extensive deformation to planes that are aligned with the fiber axis. This leads to two distinct, matrix-dominated and fiber-dominated deformation modes. Both fiber and matrix support the loads in the fiber mode, and since the fiber failure strain seldom exceeds 0.01, the total strains are small; this mode also prevails under thermal changes. However, the fiber has only a minor load sharing role in the matrix mode, hence larger strains can develop. Each mode generates a separate branch of the yield surface in the overall stress space, both branches are well described by the bimodal plasticity theory (Dvorak and Bahei-El-Din, 1987) that reliably predicts the shape and position of the initial and subsequent yield surfaces observed in combined loading experiments on thin-walled boron-aluminum tubes loaded along a complex path by combinations of axial force, internal pressure and torque (Dvorak et al., 1988; Nigam et al., 1994), and in related simulations (Dvorak et al., 1991). The ply yield stress in longitudinal shear is the only empirical parameter needed to model the matrix-dominated mode; local and overall elastic moduli are also needed in the fiber mode.

Both the aluminum matrix and composite overall yield surfaces move kinematically, by rigid body translation in the direction of the applied stress increment, except on the flat branches, where the translation is along the normal to the branch. Therefore, the loading point retains its initial position on the yield surface during continued loading. Since each point of the yield surface corresponds to a specific deformation mechanism, this implies that the initial plastic deformation mechanism stays active during subsequent plastic loading along a nonproportional but continuous path. The initial shape of the bimodal yield surface was confirmed by Ponte-Castaneda and de Botton (1992), and the underlying mechanism reproduced by numerical simulations of plastic flow in unit cells containing many aligned fibers, performed by Moulinec and Suquet (1995).

4.4. Thermal hardening

In the theory of plasticity, changes in shape and position of the yield surface are caused by mechanical loading beyond the elastic range. Thermal changes may influence the size of the yield surface, but usually not its position. However, in composite materials, thermal changes generate local stresses that superimpose with those from mechanical loading, and thus influence the onset of matrix yielding. If the temperature change is uniform in some representative volume, then its effect on internal stresses can be represented by a superposition of a certain uniform stress field supported, in part, by auxiliary surface tractions, and of the elastic field caused by removal of these tractions. This procedure leads to the conclusion that application of a uniform thermal change to a two-phase composite with an inelastic matrix causes a translation of all branches of the overall surface in the overall stress space; the result is independent of the details of microstructural geometry, and therefore is exact (Dvorak, 1997). In systems with isotropic phases, this translation is in the direction of the overall hydrostatic stress. This effect is referred to as thermal hardening. When a temperature change is applied to a laminated plate, it causes changes in both ply local stresses and ply averages, giving rise to complex, spatial, yield surface translations (Bahei-El-Din, 1992).

Of course, if the translating yield surface moves toward the current loading point, thermal changes can initiate plastic straining of the matrix. In some common metal matrix systems, temperature changes smaller than 100°C can cause local yielding. Plastic loading along a combined thermomechanical path can often be converted to a mechanical problem along a modified path by appealing to the uniform field decomposition.

4.5. Plastic deformation

An early model for inelastic deformation of fibrous systems was developed from the ideas of Adkins and Rivlin by Mulhern et al. (1967), Pipkin and Rogers (1971) and Spencer (1972). The fibers are assumed inextensible and the composite itself incompressible. The fiber dominated mode is thus suppressed, but the matrix mode is unimpeded by the presence of the stiff fibers. Agreement with the bimodal theory is obtained for deformations dominated by longitudinal or transverse shearing of the matrix (Spencer, 1992). This approach has been successful in predicting experimentally observed large deformations unidirectionally reinforced beams and tubes under monotonic loading (England and Gregory, 1995); Evans (1995) shows such comparisons for both metal and thermoplastic matrices undergoing creep deformation.

In contrast to the fairly reliable predictions of the experimentally detected yield surface shape and position, the bimodal theories, as well as other models, generally fail to simulate the inelastic strains observed in metal matrix composites under complex loading. The reason is that the direction of the overall plastic strain rate vector is not normal to the bimodal yield surface. A more fundamental reason of this absence of normality is the pronounced inhomogeneity of plastic deformation in the matrix. The Drucker (1951) stability postulate assures convexity of the yield surface and normality of the plastic strain vector in homogeneous deformation, hence it is valid locally at each material point of the deforming matrix. In the overall stress space, each such local yield surface projects a branch of the overall surface, and all active branches intersect at the loading point. Therefore, the local branches form clusters with vertices at loading points, and the experimentally detected surfaces are merely locales of these vertices; the elastic deformation region is determined by the internal envelope of the cluster. The direction of the plastic strain vector at each vertex is limited to the inside of the cone of normals to the internal envelope branches at the loading point (Hill, 1967), hence it may assume a range of directions different from the normal to the line connecting the experimentally detected yield points. Illustrations of the clusters and cones of normals, developed in a finite element simulation of actual experiments, have been presented by Dvorak et al. (1991).

The inhomogeneity of local plastic deformation and the resulting absence of normality to the apparent overall yield surface suggest the that any reliable model for incremental inelastic deformation of composite materials must be based on a fairly detailed evaluation of the internal deformation fields. This implies the use of subdivided unit cell models, analyzed with numerical schemes which can incorporate rather complex, multi-surface constitutive relations describing incremental inelastic behavior of the matrix. For fibrous systems, such models have been described, for example, by Teply and Dvorak (1988), Brockenborough et al. (1991), Dvorak et al. (1994), and Buryachenko et al. (1997); Fish et al. (1997); Buryachenko (1999). For particulate composites, where only unit-cell models deliver reasonably realistic results, relevant studies were performed by Bao et al. (1991).

Starting with the early work of Hill (1965), many models have been developed that ignore the inhomogeneity of local deformation fields and apply the inelastic constitutive relations to matrix average fields. Some were reviewed by Dvorak et al. (1998). These are easy to use, can predict reasonably well the fiber-dominated branch of the overall yield surface of fibrous plies, and may be of some value in structural applications involving only contained inelastically deforming sections. However, they underestimate plastic strains of fibrous composites subjected to complex loading. A new variant that includes direct interactions of inclusions was proposed by Ju and Tseng (1996).

Comparisons of the many proposed theories with experiments, if attempted at all, are usually limited to simple, monotonic loading regimes which can be reproduced with almost any model. Remarkably, apart from our own work (Dvorak et al., 1991) no attempt has been made to verify micromechanical model predictions by comparison with the much more discriminating combined loading experiments on fibrous systems, although Voyadjis and Thiagarajan (1996) found good agreement with a phenomenological model containing adjustable parameters.

In addition to the above approaches, variational methods for estimating effective behavior of nonlinear composites with random microstructures have been developed. These expand the early work by Hashin and Shtrikman (1962, 1963) on bounds on elastic moduli of heterogeneous solids. Generalization of the H-S principles to nonlinear elastic composites was first proposed by Talbot and Willis (1985, 1991) and Willis (1991). New variational principles using a linear comparison composite and leading to bounds for nonlinear elastic or nonlinearly viscous materials were developed by Ponte-Castaneda (1991), and by Suquet (1992). Several other procedures and more recent developments of variational methods have been reviewed and summarized by Suquet (1997) and Ponte-Castaneda and Suquet (1998). Only approximate estimates have been found for incremental theory of plasticity. More complex inelastic material behavior, especially under complex loading along a variable path, remains to be modeled in this context, but can be well represented at this time by the transformation field analysis (Dvorak, 1992; Dvorak et al., 1994) or the finite element method (Fish et al., 1997; Tvergaard, 1991; Walker et al., 1994) if, and only if, the mesh refinement allows several degrees of freedom for development of local plastic deformation in the unit cell.

4.6. Shakedown and fatigue

It is well known that plastic cyclic straining of metals leads to low-cycle fatigue crack extension. In a discontinuously reinforced matrix, the rate of crack extension may be reduced by crack bridging, or accelerated by other factors, but failure is still dominated by one or few major cracks. This is also true for composites with well-bonded aligned fibers that cannot deflect matrix cracks, such as the alumina-aluminum system. However, distributed cracks are observed in laminates where the fiber-matrix interfaces promote deflection of matrix cracks and thus encourage extension along a path aligned with the fiber axes or fiber matrix interfaces. Fairly dense but well organized crack systems can form in this manner, as observed in many polymer matrix and also in boron/aluminum and silicon carbide/titanium laminates (Bailey et al., 1979; Dvorak and Johnson, 1980; Reifsnider, 1982).

Since the fiber failure strain can be much higher than the matrix yield strain, fatigue crack systems can develop in fibrous composites and laminates under cyclic loads which are well within their overall strength range. A metal or ceramic matrix typically contributes significantly to the laminate stiffness and strength, hence these crack systems degrade the said composite properties in ways that depend on the geometry and density of the cracks in individual plies. As expected, the applied load is preferentially supported by the plies offering higher directional stiffness. Therefore, under a given load regime, fatigue damage evolution leads to redistribution of internal stresses from the weakened to the stiffer plies, and eventually to their overloading and overall failure.

Under limited amplitude loading, low-cycle fatigue damage can be avoided altogether if the laminate can reach a shakedown state through local plastic straining which restores elastic response. This can be achieved within few load cycles, well before any fatigue cracks may appear. If the load amplitude exceeds the magnitude needed for shakedown of the undamaged system, the damage-driven stress redistribution can cause unloading of the damaged plies sufficient for termination of the damage process; an apparent saturation damage state can be reached, with a reduced but essentially constant laminate stiffness. Shakedown analysis can be developed on the basis of yield surfaces and hardening rules, without a detailed knowledge of plastic flow behavior. For aligned fiber systems, this information is reliably provided by the bimodal plasticity theory, hence predictions of both initial damage envelopes and of crack densities and stiffness losses in saturated damage states are possible and have been shown to be in good agreement with experiments (Dvorak et al., 1994a). The shakedown-damage model can be easily modified for simulation of fatigue damage in polymer or ceramic matrix systems.

Damage evolution at high temperatures, the effect of matrix creep, environmental exposure of the damaged system, and possibly of other factors, remain to be resolved. Much overall progress has been made in understanding of titanium matrix systems for high temperature applications (Mall and

Nicholas, 1998). A model that includes both plasticity and damage evolution in such systems has been recently proposed by Chaboche et al. (1999).

5. Damage, delamination and fracture

By their very nature, composite materials and laminates consist of several constituents of different geometry and properties, joined along many interfaces. Both these factors contribute to several forms of local fracture events which are typically constrained from forming a major crack and are therefore nucleated under increasing load at many sites distributed through the volume of the composite material. The differences in geometry, notably between the continuous matrix and discrete reinforcement imply that long cracks may extend through the matrix. However, reinforcing fibers, particles or whiskers may not by themselves accommodate such cracks, in fact, their small diameter of few micrometers typically precludes formation of critical size Griffith cracks, even in reinforcements made of low toughness materials. This size effect is the main source of their exceptionally high strength, that can not be achieved in large volumes of the same material; high strength glass fibers provide perhaps the most striking illustration of this effect. On the next size scale, say, of a ply in a laminate, transverse cracks may extend on planes aligned with the fiber direction, but except in very brittle ceramic matrices (Evans, 1996; Hutchinson and Suo, 1992), they do not cross or break the fibers in the adjacent plies. The rate of energy release by such tunneling cracks is inversely proportional to the ply thickness, and the normal stress needed for crack extension decreases with the second power of ply thickness (Dvorak and Laws, 1986). Therefore, such cracks cannot easily extend in thin plies in polymer and metal matrix systems, but can readily multiply and become a major source of damage and stiffness reduction in laminates where ply properties and thickness allow their extension (Allen et al., 1997). In multidirectional laminates, transverse cracking is usually constrained by the adjacent plies, by possible delaminations at, or penetrations across, ply interfaces, and by interactions of adjacent crack systems at ply interfaces.

Apart from reducing stiffness and strength of laminates, transverse cracks and other damage open the material to the environment. This is of particular significance in moist or oxidating conditions. For example, polymer matrices can absorb moisture which causes volumetric changes and residual stresses. Moreover, water penetration into crack systems interferes with crack opening and closing under cyclic loading, and degrades endurance; it may also contribute to fiber-matrix debonding. These and other aspects of hygrothermal and damage response of polymeric composites have been studied by Weitsman and associates (Smith and Weitsman, 1996; Weitsman, 2000).

Differences in elastic properties of the constituents also play a role in damage evolution. As Erdogan points out in his chapter, cracks approaching interfaces between bonded solids are attracted by regions of low stiffness. Therefore, crack bridging can be promoted by compliant particles that are well bonded to a stiff matrix. Particles that dilate due to a stress-induced phase transformation ahead of a crack can generate a crack arresting compressive stress (Amazigo and Budiansky, 1988).

Interfaces may impede or promote damage development. On one hand, they may serve to deflect matrix cracks from the reinforcement or from adjacent plies, and thus allow operation of crack bridging processes that impose a major constraint on extension of matrix cracks. On the other hand, if exposed to high peeling and shear stresses, they may separate and thus open interfacial cracks between matrix and reinforcements, or delaminations between plies inside and at free edges of laminates (Pagano, 1978; Zheng and Sun, 1998; Quian and Sun, 1998). Delamination between plies is of major concern in all loaded plates and shells, where it may facilitate buckling leading to catastrophic failure (Kardomateas and Pelegri, 1994).

Together with analytical tools that help prevent or control evolution of damage, a major line of defense against its adverse effects is offered by non-destructive inspection methods for evaluation of structural integrity. Major advances in this field are described by Jan Achenbach in this volume. Related research into *in-situ* ultrasonic measurement of composite elastic properties and detection of interface flaws has been reported by Sachse (Baker and Sachse, 1996).

Reinforcement by long fibers, and aligned fibers in particular, typically elevates toughness of the matrix material and impedes growth of dominant cracks. Therefore, premature macroscopic fracture of fiber systems is likely to initiate from geometrical imperfections, such as holes and notches which serve some useful function or result from impact damage. Macroscopic fracture is typically preceded by local inelastic deformation or damage. Therefore, direct application of standard linear fracture mechanics is not indicated. However, reliable fracture strength predictions of fibrous laminates can be derived from a criterion that requires a certain critical normal stress magnitude to be reached within a small process zone ahead of the notch. As Fazil Erdogan notes in his chapter in this volume, a criterion of this kind was first introduced by Weighardt in 1907. Its validity for small scale inelastic deformation or damage in polymer matrix systems was demonstrated by Whitney and Nuismer (1974), with stresses derived from available anisotropic elastic crack solutions, and the process zone found by fitting predictions to measured notched strengths. Experimental evaluations were carried out by Daniel, among others, and reexamined recently by Bazant et al. (1996) in the context of a size effect model.

Large scale yielding or delamination may develop at notch tips in systems with very ductile metal or polymer matrices, in the form of long, discrete plastic or damage zones aligned with the fiber direction. Fracture is preceded localized fiber breaks, crack bridging and matrix stretching at a small distance ahead of the notch tip. This process is consistent with the assumed existence of a highly stressed process zone where fracture initiates, but is often analyzed without regard to the actual stress field, dominated now by the logarithmic singularity associated with the long plastic zones. When the actual field is taken into consideration, the critical stress criterion generally predicts results of numerous experiments (Dvorak et al., 1992). These long plastic zones cause departure from fracture mechanics, in the sense that the onset of fracture cannot be characterized by a single parameter such as a stress intensity factor; a different scaling procedure is required for application of test data to different notch geometries.

Dynamic loading of composite materials is of concern in many applications, especially in those involving large impact loads supported by thick laminated sections. Development of dynamic constitutive relations for composites is among the current challenges. Both tensile and compressive strength of polymer matrix systems, such as E-glass/vinyl ester, increase with strain rate, and also with lateral confinement. Damage modes may change as well, for example, from splitting to kinking in compression, and become more varied under different loads and confinements. These variations can be traced to the presence of numerous interfaces which provide weak or preferred directions for initiation of cracks and for their subsequent growth. Some of these issues are discussed by Rosakis in this volume. Impact and indentation resistance of laminates has been explored by Sun and associates, in several studies over the years (e. g., Sun and Jih, 1995; Weeks and Sun, 1998). High strain rate effects are extensively explored in Vinson and Rajapakse (1995), also by Li and Weng (1994).

6. Research needs

(i) The general area of deformation, damage initiation and growth, and failure of composite materials and laminated and sandwich structures subjected to complex loading conditions in severe environments requires much attention in future research. Physically based quantitative theories and models with predictive capabilities are needed. The entire life cycle of the structure or part has to be taken into consideration, from the fabrication and processing phase, through evolution of elastic and inelastic and/or damage-assisted deformation, to final failure. Different spatial scales, spanning the microstructure of fibers, coatings, matrices, plies, and finally large structures have to be modeled, together with different temporal scales from dynamic short-term loads to long term creep and life prediction on the scale of several decades. Interactions of short-term loads with sustained loads applied by gravity or water pressure need to be explored. These results are needed to support development of thick composites structures for infrastructure, marine and ship structures, and similar applications.

(ii) Recent advances in our ability to design physical properties of composite material systems and structures for different specific purposes or missions needs to be exploited in future structural designs. Such 'active modeling' employs the available knowledge base from research on existing systems and structures, to suggest design and processing measures that yield not only desirable stiffness and strength values, but also provide property and performance improvements. For example, judicious application of fiber prestress to reduce fiber waviness can improve compressive strength and, at the same time, such prestress can be utilized in residual stress management and damage retardation in the plies and at free edges of laminates. These and other fabrication methods need to be modeled and controlled to minimize the density of preexisting flaws and to improve initial stress states.

(iii) Resistance of composite and sandwich structures to dynamic and low impact loads and under high loading rates needs to be improved both by experimentally supported modeling and by selective reinforcement that improves adhesion along ply and other interfaces. Residual stresses that promote coherence of interfaces should be generated where possible. Various global-local approaches to modeling of deformation and damage evolution in complex laminates should be expanded in support of these efforts. Environmental effects caused by moist or hot environments need to be contained at structural surfaces and prevented from degrading the properties and integrity of the composite material inside the structure.

(iv) Metal and ceramic composites appear to have the best potential as materials for high temperature applications. Novel material combinations and fabrication methods need to be identified and explored to achieve performance and life expectancy criteria at temperatures well above 1000 C.

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